Class- X Session- 2022-23

Subject- Mathematics (Standard)

Sample Question Paper - 4

with Solution

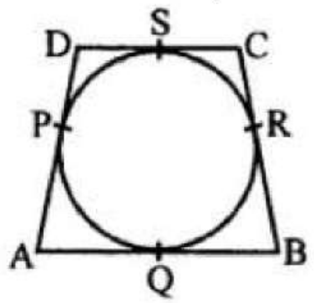
Time Allowed: 3 Hrs. **Maximum Marks : 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section **A** has 20 MCQs carrying 1 mark each
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. In the given figure, quad. ABCD is circumscribed, touching the circle at P, Q, R and S. If AP = 5 cm, BC = 7 cm and CS = 3 cm. Then, the length AB = ? **[1]**



a) 8 cm b) 9 cm

c) 10 cm d) 12 cm

2. A line intersects the y-axis and x-axis at the points P and Q, respectively. If (2, –5) is the mid-point of PQ, then the coordinates of P and Q are, respectively **[1]**

a) (0, – 5) and (2, 0) b) (0, 4) and (– 10, 0)

c) (0, 10) and (– 4, 0) d) (0, – 10) and (4, 0)

3. The distance between the points (a cos 25°, 0) and (0, a cos 65°) is **[1]**

a) None of these b) 3a

c) a d) 2a

4. A die is thrown once. The probability of getting an odd number greater than 3 is **[1]**

a) $\frac{1}{3}$ b) $\frac{1}{6}$

c) 0

d) $\frac{1}{2}$

5. The ratio in which the line segment joining points $A(a_1, b_1)$ and $B(a_2, b_2)$ is divided by y-axis is **[1]**

a) $a_1 : a_2$

b) $-a_1 : a_2$

c) $-b_1 : b_2$

d) $b_1 : b_2$

6. If $\frac{2x+y+2}{5} = \frac{3x-y+1}{3} = \frac{3x+2y+1}{6}$ then **[1]**

a) x = 2, y = 1

b) x = -1, y = -1

c) x = 1, y = 1

d) x = 1, y = 2

7. A solid consists of a circular cylinder with an exact fitting right circular cone placed at the top. The height of the cone is h. If the total volume of the solid is 3 times the volume of the cone, then the height of the circular cylinder is **[1]**

a) 4h

b) 2h

c) h

d) $\frac{2h}{3}$

8. A box contains cards numbered 6 to 50. A card is drawn at random from the box. The probability that the drawn card has a number which is a perfect square is **[1]**

a) $\frac{1}{9}$

b) $\frac{4}{45}$

c) $\frac{2}{15}$

d) $\frac{1}{45}$

9. If x = 3 is a solution of the equation $3x^2 + (k - 1)x + 9 = 0$ then k = ? **[1]**

a) 13

b) -11

c) 11

d) -13

10. The probability of guessing the correct answer to certain text questions is $\frac{x}{12}$. If the probability of not guessing the answer is $\frac{5}{8}$, then the value of x is **[1]**

a) 1

b) 0

c) 4

d) 4.5

11. If $\sqrt{3}\tan 2\theta - 3 = 0$ then $\theta = ?$ **[1]**

a) 30^o

b) 60^o

c) 15^o

d) 45^o

12. If two positive integers m and n can be expressed as $m = x^2y^5$ and $n = x^3y^2$, where x and y are prime numbers, then HCF(m, n) = **[1]**

a) x^2y^2

b) x^2y^3

c) x^3y^2

d) x^3y^3

13. A two digit number is such that the product of the digits is 12. When 36 is added to the number then the digits interchange their places. The number is **[1]**

a) 26

b) 34

c) 43

d) 62

14. If two trees of height 'x' and 'y' standing on the two ends of a road subtend angles of $30^\circ$and 60° respectively at the midpoint of the road, then the ratio of x : y is **[1]**

a) 1 : 3

b) 1 : 2

c) 3 : 1

d) 1 : 1

15. If the point (x, 4) lies on a circle whose centre is at the origin and radius is 5 then x = **[1]**

a) 0

b) ±3

c) ±4

d) ±5

16. If $\frac{241}{4000} = \frac{241}{2^m \times 5^n}$, then **[1]**

a) m = 3 and n = 2

b) m = 5 and n = 3

c) m = 2 and n = 5

d) m = 4 and n = 5

17. The wickets taken by a bowler in 10 cricket matches are 2, 6, 4, 5, 0, 3, 1, 3, 2, 3. The median of the data is **[1]**

a) 3

b) 1

c) 2.5

d) 2

18. **Assertion (A):** H.C.F. of 12 and 77 is 1. **[1]**
Reason (R): L.C.M. of two coprime numbers is equal to their product.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

19. The pair of equations x = a and y = b graphically represents lines which are **[1]**

a) intersecting at (b, a)

b) coincident

c) intersecting at (a, b)

d) parallel

20. **Assertion (A):** If two triangles are similar then they are congruent also. **[1]**
Reason (R): Ratio of perimeters of two triangles is always equal to ratio of their

corresponding sides, medians, altitudes and angle bisectors.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Is the system of linear equations 4x + 6y = 18 and 2x + 3y = 9 consistent? Justify your answer. **[2]**

22. In a cricket match, a batsman hits a boundary 6 times out of 30 balls he plays. Find the probability that on a ball played: **[2]**

i. he hits boundary

ii. he does not hit a boundary.

23. Name the type of triangle PQR formed by the points P $(\sqrt{2}, \sqrt{2})$, Q $(-\sqrt{2}, -\sqrt{2})$ and R $(-\sqrt{6}, \sqrt{6})$. **[2]**

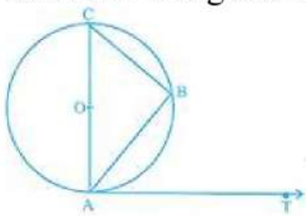
OR

Find the centroid of the triangle whose vertices are (4, -8) (-9, 7) and (8, 13).

24. Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at the centre. **[2]**

OR

If AB is a chord of a circle with centre O. AOC is a diameter and AT is the tangent at A as shown in figure. Prove that ∠BAT = ∠ACB.



25. Can the quadratic polynomial $x^2 + kx + k$ have equal zeroes for some odd integer k > 1? Justify. **[2]**

Section C

26. Solve the following system of linear equations: 35x + 23y = 209; 23x + 35y = 197. **[3]**

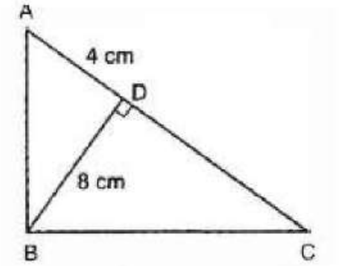
27. If tan A = n tanB and sin A = m sinB, then prove that $\cos^2 A = \frac{m^2-1}{n^2-1}$ **[3]**

28. Prove that $(3 + 2\sqrt{5})^2$ is irrational. **[3]**

OR

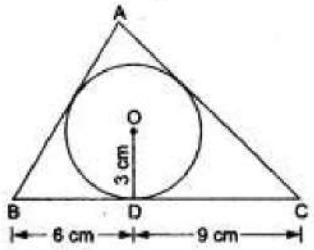
Show that $3 + 5\sqrt{2}$ is an irrational number.

29. In Fig. ∠ABC= 90° and $BD \perp AC$. If BD = 8 cm and AD= 4 cm, find CD. **[3]**



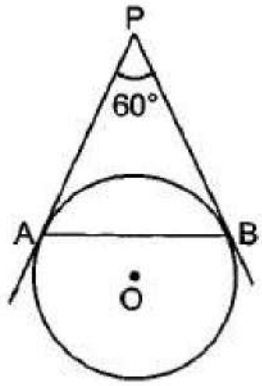
30. The lower window of a house is at a height of 2 m above the ground and its upper window is 4 m vertically above the lower window. At certain instant the angles of elevation of a balloon from these windows are observed to be 60° and 30° respectively. Find the height of the balloon above the ground. **[3]**

31. In the given figure, a triangle ABC is drawn to circumscribe a circle of radius 3 cm such that the segments BD and DC into which BC is divided by the point of contact D, are of lengths 6 cm and 9 cm respectively. If the area of $\triangle ABC = 54$ cm then find the lengths of sides AB and AC. **[3]**



OR

In figure, AP and BP are tangents to a circle with centre O, such that AP = 5 cm and ∠APB = 60° Find the length of chord AB.



Section D

32. ABCD is a trapezium in which AB || DC and P and Q are points on AD and BC, respectively such that PQ || DC. If PD = 18 cm, BQ = 35 cm and QC = 15 cm, find AD. **[5]**

33. A man travels a distance of 300 km at a uniform speed. If the speed of the train is increased by 5 km an hour, the journey would have taken two hours less. Find the original speed of the train. **[5]**

OR

A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 km/h more than the first speed. If it takes 3 hours to complete the total journey, what is its first speed?

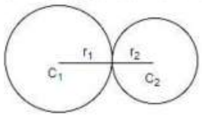
34. Calculate the mode of the following frequency distribution table : **[5]**

Marks	**Number of students**
25 or more than 25	52
35 or more than 35	47
45 or more than 45	37
55 or more than 55	17
65 or more than 65	8
75 or more than 75	2
85 or more than 85	0

35. Find the area of the segment of a circle of radius 12 cm whose corresponding sector central angle 60°. (Use $\pi = 3.14$). **[5]**

OR

Two farmers have circular plots. The plots are watered with the same water source placed in the point common to both the plots as shown in the figure. The sum of their areas is 130π and the distance between their centres is 14 m.Find the radii of the circles.What value is depicted by the farmers?



Section E

36. **Read the text carefully and answer the questions:** **[4]**

Kamla and her husband were working in a factory in Seelampur, New Delhi. During the pandemic, they were asked to leave the job. As they have very limited resources to survive in a metro city, they decided to go back to their hometown in Himachal Pradesh. After a few months of struggle, they thought to grow roses in their fields and sell them to local vendors as roses have been always in demand. Their business started growing up and they hired many workers to manage their garden and do packaging of the flowers.

In their garden bed, there are 23 rose plants in the first row, 21 are in the 2^{nd}, 19 in 3^{rd} row and so on. There are 5 plants in the last row.

(i) How many rows are there of rose plants?

(ii) Also, find the total number of rose plants in the garden.

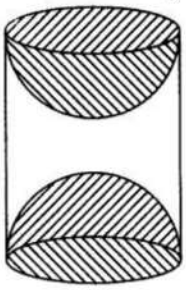
OR

If total number of plants are 80 in the garden, then find number of rows?

(iii) How many plants are there in 6th row.

37. **Read the text carefully and answer the questions:** **[4]**

A carpenter used to make different kinds and different shapes of a toy of wooden material. One day a man came to his shop to purchase an article that has values as per his requirement. He instructed the carpenter to make the toy by taking a wooden block of rectangular shape with height 12 cm and width 9 cm, then shaping this block as a solid cylinder and then scooping out a hemisphere from each end, as shown in the given figure. The difference between the length of rectangle and height of the cylinder is 2 cm (Rectangle length > Cylinder height), and the difference between the breadth of rectangle and the base of cylinder is also 2 cm (Rectangle breadth > Cylinder base(diameter)).



(i) Find the volume of the cylindrical block before the carpenter started scooping the hemisphere from it.

(ii) Find the volume of wood scooped out?

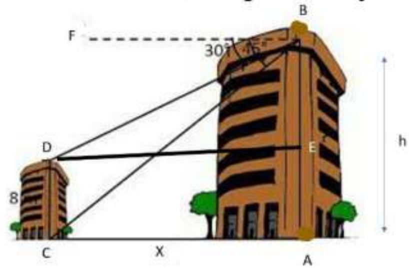
(iii) Find the total surface area of the article?

OR

Find the total surface area of cylinder before scooping out hemisphere?

38. **Read the text carefully and answer the questions:** **[4]**

Basant and Vinod lives in a housing society in Dwarka, New Delhi. There are two building in their housing society. The first building is 8 meter tall. One day, both of them were just trying to guess the height of the other multi-storeyed building. Vinod said that it might be a 45 degree angle from the bottom of our building to the top of multi-storeyed building so the height of the building and distance from our building to this multi-storeyed building will be same. Then, both of them decided to estimate it using some trigonometric tools. Let's assume that the first angles of depression of the top and bottom of an 8 m tall building from top of a multi-storeyed building are 30^{o} and 45^{o}, respectively.



(i) Now help Vinod and Basant to find the height of the multistoried building.

(ii) Also, find the distance between two buildings.

OR

Find the distance between top of multistoried building and top of first building.

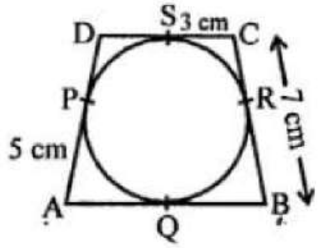
(iii) Find the distance between top of multistoried building and bottom of first building.

Solution

Section A

1. **(b)** 9 cm

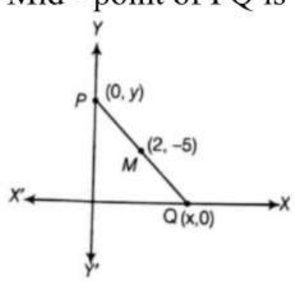
Explanation: In the figure, quadrilateral ABCD is circumscribed touches the circle at P, Q, R and S



AP = 5 cm, BC = 7 cm, CS = 3 cm AB = ?
Tangents drawn from the external point to the circle are equal
AQ = AP = 5 cm
CR = CS = 3 cm
BQ = BR
Now, BR = BC – CR = 7 – 3 = 4 cm
BQ = 4 cm
Now, AB = AQ + BQ = 5 + 4 = 9 cm

2. **(d)** (0, – 10) and (4, 0)

Explanation:
Let the coordinates of P (0, y) and Q (x, 0).
So, the mid - point of P (0, y) and Q (x, 0) = M
Coordinates of M = $\left(\frac{0+x}{2}, \frac{y+0}{2}\right)$
∴ Mid - point of a line segment having points (x_1, y_1) and (x_2, y_2)
$= \left(\frac{(x_1+x_2)}{2}, \frac{(y_1+y_2)}{2}\right)$
Given,
Mid - point of PQ is (2, - 5)



$\therefore 2 = \frac{x+0}{2}$ = 4 = x + 0
x = 4
$-5 = \frac{y+0}{2}$ = - 10 = y + 0
- 10 = y
So,
x = 4 and y = - 10
Thus, the coordinates of P and Q are (0, - 10) and (4, 0)

3. **(c)** a

Explanation: Distance between (a cos 25^{o}, 0) and (0, a cos65^{o})
$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(0 - a\cos 25^\circ)^2 + (a\cos 65^\circ - 0)^2}$
$= \sqrt{a^2\cos^2 25^\circ + a^2\cos^2 65^\circ}$
$= \sqrt{a^2[\cos^2 25^\circ + \cos^2 65^\circ]}$
$= a\sqrt{\cos^2(90^\circ - 65^\circ) + \cos^2 65^\circ}$
$= a\sqrt{\sin^2 65^\circ + \cos^2 65^\circ}$
$= a(\sqrt{1}) = a$

4. **(b)** $\frac{1}{6}$

Explanation: Number of all possible outcomes = 6.
Odd number greater than 3 is 5 only. Its number is 1.
$\therefore$ P(getting an odd number greater than 3) = $\frac{1}{6}$

5. **(b)** -a_1 : a_2

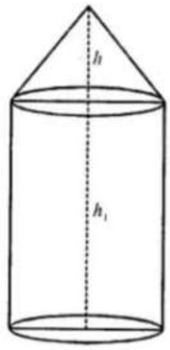
Explanation: Let the point P on y-axis, divides the line segment joining the point A(a_1, b_1 and B(a_2, b_2) is the ratio m_1: m_2 and
let the co-ordinates of P be (0, y), then
$0 = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} \Rightarrow 0 = \frac{m_1a_2 + m_2a_1}{m_1 + m_2}$
$\Rightarrow m_1a_2 + m_2a_1 = 0 \Rightarrow m_1a_2 = -m_2a_1$
$\Rightarrow \frac{m_1}{m_2} = \frac{-a_1}{a_2}$
Ratio is -a_1 : a_2

6. **(c)** x = 1, y = 1

Explanation: $\frac{2x+y+2}{5} = \frac{3x-y+1}{3}$
$\Rightarrow$ 6x + 3y + 6 = 15x - 5y + 5 $\Rightarrow$ 9x - 8y = 1 .....(i)
$\frac{3x-y+1}{3} = \frac{3x+2y+1}{6}$
$\Rightarrow$ 18x - 6y + 6 = 9x + 6y + 3 $\Rightarrow$ 9x - 12y = - 3 .....(ii)
Solve (i) and (ii) to get x = 1 and y = 1.

7. **(d)** $\frac{2h}{3}$

Explanation:
Height of cone = h
Volume of solid = 3 $\times$ volume of cone
Let h be the height of the cylinder and r be its radius, then
Volume of cylinder and r be its radius, then
Volume of cylinder = $\pi r^2 h_1$
and volume of cone = $\left(\frac{1}{3}\right)\pi r^2 h_1$



Then volume of solid = $\pi r^2 h_1 + \frac{1}{3}\pi r^2 h$
$= \pi r^2\left(h_1 + \frac{1}{3}h\right)$
Now $\pi r^2\left(h_1 + \frac{1}{3}h\right) = 3 \times \frac{1}{3}\pi r^2 h = \pi r^2 h$
$\Rightarrow h_1 + \frac{1}{3}h = h$ (comparing)

$h_1 = h - \frac{1}{3}h = \frac{2}{3}h$

Hence, height of cylinder = $\frac{2h}{3}$

8. **(a)** $\frac{1}{9}$

Explanation: Given numbers are 6, 7,8,9,..., 50

Number of these numbers = 50 - 5 = 45

Perfect square numbers from these are 3^2, 4^2 ,5^2 , 6^2, 7^2

Their number is 5.

$\therefore$ P(getting a perfect square number) = $\frac{5}{45} = \frac{1}{9}$

9. **(b)** -11

Explanation: $3x^2 + (k-1)x + 9 = 0$

x = 3 is a solution of the equation means it satisfies the equation

Put x = 3, we get

$3(3)^2 + (k - 1)\, 3 + 9 = 0$

27 + 3 k - 3 + 9 = 0

27 + 3 k + 6 = 0

3 k = - 33

k = - 11

10. **(d)** 4.5

Explanation: Since, Total Probability = 1

$\therefore \frac{x}{12} + \frac{5}{8} = 1$

$\Rightarrow \frac{2x+15}{24} = 1$

$\Rightarrow$ 2x + 15 = 24

$\Rightarrow$ 2x = 9

$\Rightarrow$ x = 4.5

11. **(a)** 30°

Explanation: $\sqrt{3}\tan 2\theta - 3 = 0$

$\Rightarrow \sqrt{3}\tan 2\theta = 3$

$\Rightarrow \tan 2\theta = \frac{3}{\sqrt{3}}$

$\Rightarrow \tan 2\theta = \sqrt{3}$

$\Rightarrow \tan 2\theta = \tan 60^\circ$

$\Rightarrow 2\theta = 60^\circ$

$\Rightarrow \theta = 30^\circ$

12. **(a)** x^2y^2

Explanation: $x^2y^5 = y^3(x^2y^2)$

$x^3y^3 = x(x^2y^2)$

Therefore HCF (m, n) is x^2y^2

13. **(a)** 26

Explanation: Let the digit at the units place be x.

The digit at the tens place is $\frac{12}{x}$.

Original Number = 10x$\frac{12}{x}$+x = $\frac{120}{x}$+x

Reverse Number = 10 x x + $\frac{12}{x}$ = $10x + \frac{12}{x}$.

Now, Reverse Number = Original Number + 36

=> $10x + \frac{12}{x} = \frac{120}{x} + x + 36$

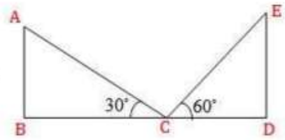
=> $9x + \frac{12}{x} - \frac{120}{x} - 36 = 0$

=> $\frac{9x^2 - 108 - 36x}{x} = 0$

$\Rightarrow 9x^2 - 36x - 108 = 0$
$\Rightarrow x^2 - 4x - 12 = 0$
$\Rightarrow x^2 - 6x + 2x - 12 = 0$
$\Rightarrow x(x-6) + 2(x-6) = 0$
$\Rightarrow (x-6)(x+2) = 0$
$\Rightarrow x - 6 = 0$ or $x + 2 = 0$
$\Rightarrow x = 6$ or $x = -2$ (But x cannot be -2)
Digit at the units place = 6
Digit at the tens place = $\frac{12}{6}$ =2
Thus, the original number = 26

14. **(a)** 1 : 3

Explanation:



Here two trees AB and ED are of height x x and y y respectively. And BC = CD
$\therefore \tan 30^\circ = \frac{x}{\text{BC}}$
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{\text{BC}}$
$\Rightarrow x = \frac{\text{BC}}{\sqrt{3}}$ And $\tan 60^\circ = \frac{y}{\text{CD}}$
$\Rightarrow \sqrt{3} = \frac{y}{\text{CD}}$
$\Rightarrow y = \text{CD}\sqrt{3} = \text{BC}\sqrt{3}$ [BC = CD]
Now, $\frac{x}{y} = \frac{\text{BC}}{\sqrt{3} \times \text{BC}\sqrt{3}}$
$= \frac{1}{3}$
$\Rightarrow x : y = 1 : 3$

15. **(b)** ± 3
Explanation: Point A(x, 4) is on a circle with centre O(0, 0) and radius = 5
$\therefore \text{OA} = \sqrt{(x-0)^2 + (4-0)} = \sqrt{x^2 + 16}$
$\therefore \sqrt{x^2+16} = 5 \Rightarrow x^2 + 16 = 25$
Squaring on both sides, we get
$\Rightarrow x^2 = 25 - 16 = 9 = (\pm 3)^2$
$\therefore x = \pm 3$

16. **(b)** m = 5 and n = 3
Explanation: $\frac{241}{4000} = \frac{241}{2^m \times 5^n}$
$\Rightarrow \frac{241}{2^5 \times 5^3} = \frac{241}{2^m \times 5^n}$
Comparing the denominators of both fractions, we have m = 5 and n = 3

17. **(a)** 3
Explanation: Arranging the given data in ascending order,
0, 1, 2, 2, 3, 3, 3, 4, 5, 6
Here n = 10, which is even.
$\therefore$ Median = $\frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} \text{term} + \left(\frac{n}{2}+1\right)^{th} \text{term}\right]$
$= \frac{1}{2}\left[5^{th} \text{ term} + 6^{th} \text{ term}\right]$
$\Rightarrow \frac{1}{2}[3+3] = \frac{6}{2}$
= 3

18. **(b)** Both A and R are true but R is not the correct explanation of A.
Explanation: Yes 12 and 17 are coprime numbers and H.C.F. of coprimes is always 1.

19. **(c)** intersecting at (a, b)
Explanation: The pair of equations x = a and y = b graphically represents lines which are

intersecting at (a, b), as when we plot the graphs x=a will run parallel to y-axis and y=b will run parallel to x-axis but they will intersect each other at point (a,b)

20. **(d)** A is false but R is true.
Explanation: Similar triangles are not always congruent.

Section B

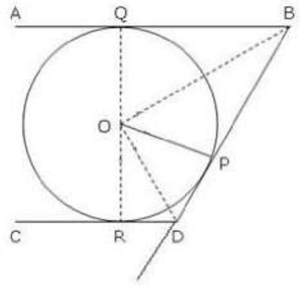
21. The given equations are
$4x + 6y = 18$
So, $4x + 6y - 18 = 0$ .... (i)
And $2x + 3y = 9$
So, $2x + 3y - 9 = 0$ .... (ii)
The given equations are in the form of
$a_1x + b_1 y + c_1 = 0$
and $a_2x + b_2y + c_2 = 0$
After comparing, we get
$a_1 = 4, b_1 = 6, c_1 = -18$
$a_2 = 2, b_2 = 3, c_2 = -9$
It can be observed that:
$\frac{a_1}{a_2} = \frac{4}{2} = 2$
$\frac{b_1}{b_2} = \frac{6}{3} = 2$
$\frac{c_1}{c_2} = \frac{-18}{-9} = 2$
$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
Thus, the given system of equations has infinitely many solutions and thus is consistent.

22. Number of times batsman hits a bondary = 6
Total number of balls played = 30
$\therefore$ Number of times that the batsman does not hit a boundary = 30 - 6 = 24
i. P(he hits a boundary) = $\frac{Number\ of\ times\ when\ he\ hits\ boundary}{Total\ number\ of\ balls\ played} = \frac{6}{30} = \frac{1}{5}$
ii. P(he does not hit a boundary) = $\frac{Number\ of\ times\ when\ he\ not\ hits\ boundary}{Total\ number\ of\ balls\ played} = \frac{24}{30} = \frac{4}{5}$

23. $P(\sqrt{2}, \sqrt{2}), Q(-\sqrt{2}, -\sqrt{2})$ and $R(-\sqrt{6}, \sqrt{6})$ are the vertices of ΔPQR.
Now,
$PQ = \sqrt{(-\sqrt{2} - \sqrt{2})^2 + (-\sqrt{2} - \sqrt{2})^2} = \sqrt{(-2\sqrt{2})^2 + (-2\sqrt{2})^2} =$
$= \sqrt{8 + 8} = \sqrt{16} = 4\ units$
$QR = \sqrt{(-\sqrt{6} + \sqrt{2})^2 + (\sqrt{6} + \sqrt{2})^2} = \sqrt{6 + 2 - 2\sqrt{12} + 6 + 2 + 2\sqrt{12}} = \sqrt{16} = 4\ units$
$PR = \sqrt{(-\sqrt{6} - \sqrt{2})^2 + (\sqrt{6} - \sqrt{2})^2} = \sqrt{6 + 2 + 2\sqrt{12} + 6 + 2 - 2\sqrt{12}} = \sqrt{16} = 4\ units$
Since PQ = QR = PR, ΔPQR is an equilateral triangle.

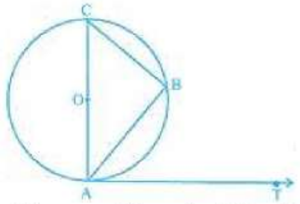
OR

Let (x, y) be the coordinate of centroid
$x = \frac{x_1 + x_2 + x_3}{3}$
$= \frac{4 - 9 + 8}{3} = \frac{3}{3} = 1$
$y = \frac{y_1 + y_2 + y_3}{3}$
$= \frac{-8 + 7 + 13}{3} = \frac{20 - 8}{3} = 4$
Coordinate of centroid is (1, 4)

24. 

Given: AB and CD are two parallel tangents, another tangent BD intersects them at B and D respectively. The intercept BD subtends ∠BOD at center O.
To prove: ∠BOD = 90°
Proof: In △BOP and △BOQ,
OP = OQ = radius
OB is common
BP = BQ (Tangents from one point B)
So △BOP ≅ △ BOQ (By SSS criteria)
Hence ∠OBP = ∠OBQ
So ∠QBP = 2∠OBP........(1)
Similarly in △DOP and △DOR,
∠ODP = ∠ODR
and ∠RDP = 2∠ODP ......(2)
Now, $AB \parallel CD$ and BD is a transversal line.
So ∠QBP +∠RDP = 180° (The interior angles formed on the same side of the transversal line)
From eqn (1) and (2),
2 ∠OBP + 2 ∠ODP = 180°
So ∠OBP + ∠ODP = 90°
Now in △BOD,
∠BOD + ∠OBP + ∠ODP = 180°
∠BOD + 90 = 180°
Therefore ∠BOD = 90°
Hence proved.

OR



Given: Chord AB, diameter AOC and tangents at A of a circle with centre O.
To prove: ∠BAT = ∠ACB
Proof: Radius OA and tangent AT at A are perpendicular.
∴ ∠OAT = 90^o (radius at the point of contact of tangent is perpendicular)
⇒ ∠BAT = 90^o - ∠BAC .....(i)
AOC is diameter.
∴ ∠B = 90
⇒ ∠C + ∠BAC = 90^o
⇒ ∠C = 90^o - ∠BAC ...(ii)
From (i) and (ii), we get
∠BAT = ∠ACB. Hence, proved.

25. NO
For the roots to be equal in,

$x^2 + kx + k$,
we must have the discriminant zero.
i.e. $k^2 - 4k = 0$
$k = 4$.

Section C

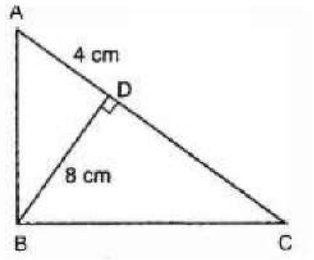
26. The given system of equations is
$35x + 23y = 209$ ....(1)
$23x + 35y = 197$ ....(2)
Adding equation (1) and equation (2), we get
$58x + 58y = 406$
$\Rightarrow x + y = 7$ ....(3) ....Dividing throughout by 58
Subtracting equation (2) from equation (1), we get
$12x - 12y = 12$
$\Rightarrow x - y = 1$ .....(4) ....Dividing throughout by 12
Adding equation (3) and equation (4), we get $2x = 8$
$\Rightarrow \; x = \frac{8}{2} = 4$
Subtracting equation (4) from equation (3), we get $2y = 6$
$\Rightarrow \; y = \frac{6}{2} = 3$
$\Rightarrow$ hence, the solution of the given pair of equations is
$x = 4, y = 3$.
Verification : Substituting $x = 4, y = 3$
We find that both the equations (1) and (2) are satisfied as shown below:
$35x + 23y = 35(4) + 23(3) = 140 + 69 = 209$
$23x + 35y = 23(4) + 35(3) = 92 + 105 = 197$
Hence, the solution is correct.
27. Given,
$\tan A = n \tan B$
$\Rightarrow \tan B = \frac{1}{n} \tan A$
$\Rightarrow \cot B = \frac{n}{\tan A}$..........(1)
Also given,
$\sin A = m \sin B$
$\Rightarrow \sin B = \frac{1}{m} \sin A$
$\Rightarrow \operatorname{cosec} B = \frac{m}{\sin A}$.....(2)
We know that, $\operatorname{cosec}^2 B - \cot^2 B = 1$, hence from (1) & (2) :-

$\frac{m^2}{\sin^2 A} - \frac{n^2}{\tan^2 A} = 1$
$\Rightarrow \quad \frac{m^2}{\sin^2 A} - \frac{n^2 \cos^2 A}{\sin^2 A} = 1$
$\Rightarrow \quad \frac{m^2 - n^2 \cos^2 A}{\sin^2 A} = 1$
$\Rightarrow m^2 - n^2\cos^2 A = \sin^2 A$
$\Rightarrow m^2 - n^2\cos^2 A = 1 - \cos^2 A$
$\Rightarrow m^2 - 1 = n^2\cos^2 A - \cos^2 A$
$\Rightarrow m^2 - 1 = (n^2 - 1)\cos^2 A$
$\Rightarrow \quad \frac{m^2 - 1}{n^2 - 1} = \cos^2 A$
28. Let take that $3 + 2\sqrt{5}$ is a rational number.
So we can write this number as

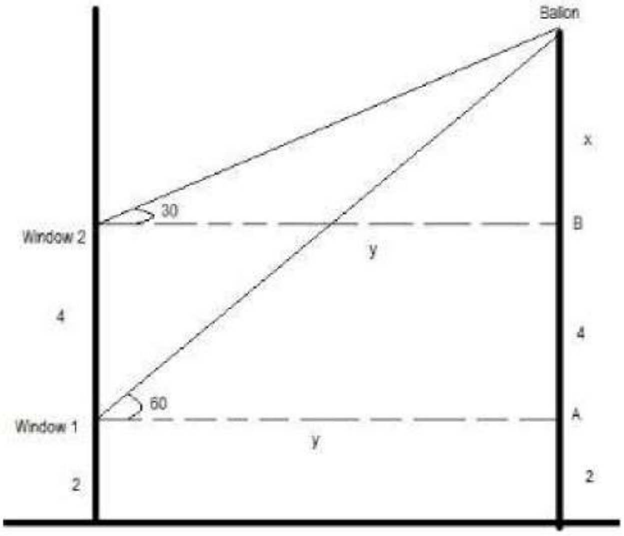
$3 + 2\sqrt{5} = \frac{a}{b}$
Here a and b are two co-prime numbers and b is not equal to 0.
Subtract 3 both sides we get,
$2\sqrt{5} = \frac{a}{b} - 3$
$2\sqrt{5} = \frac{a-3b}{b}$
Now divide by 2 we get
$\sqrt{5} = \frac{a-3b}{2b}$
Here a and b are an integer so $\frac{a-3b}{2b}$ is a rational number so $\sqrt{5}$ should be a rational number but $\sqrt{5}$ is an irrational number so it contradicts the fact.
Hence the result is $3 + 2\sqrt{5}$ is an irrational number
Now its square will again contain an irrational number.
Hence the given number is an irrational number.

OR

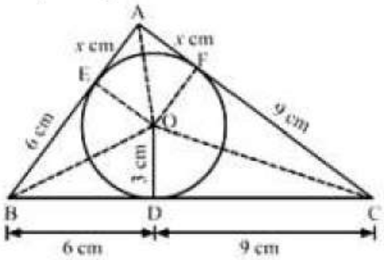
Let $3 + 5\sqrt{2}$ be rational and have only common factor 1.
Let, $3 + 5\sqrt{2} = \frac{a}{b}$
$5\sqrt{2} = \frac{a}{b} - 3$
$\sqrt{2} = \frac{a-3b}{5b}$
If $\frac{a-3b}{5b}$ is rational so, $\sqrt{2}$ is also rational number but it is not true as $\sqrt{2}$ is an irrational number.
So it is contradiction to our assumption,
Therefore, $3 + 5\sqrt{2}$ is an irrational number.

29. 

According to the question, $\angle ABC = 90^\circ$ and $BD \perp AC$
Now, $\angle ABD + \angle DBC = 90^\circ$ ....(i) [$\because \angle ABC = 90^\circ$]
And, $\angle C + \angle DBC = 90^\circ$ ....(ii) [By angle sum prop. in ΔBCD]
On comparing equations (i) and (ii), we get
$\angle ABD = \angle C$ ....(iii)
Referring to the given figure, we observe that in ΔABD and ΔBCD,
$\angle ABD = \angle C$ [From (iii)]
$\angle ADB = \angle BDC$ [Each 90°]
Then, ΔABD ~ ΔBCD [By AA similarity]
$\therefore \frac{BD}{CD} = \frac{AD}{BD}$ [Corresponding parts of similar Δ are proportional]
$\Rightarrow \frac{8}{CD} = \frac{4}{8}$
$\Rightarrow CD = \frac{8 \times 8}{4} = 16cm$

30. 

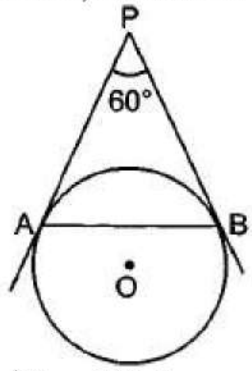
From the figure,
let the height of balloon is = x+4+2 = x+6
$\tan 30^\circ = \frac{x}{y}$
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{y}$
$\Rightarrow y = \sqrt{3}x$ .... (i)
And, $\tan 60^\circ = \frac{x+4}{y}$
$\Rightarrow \tan 60^\circ = \frac{x+4}{y}$
$\Rightarrow \sqrt{3} = \frac{x+4}{y}$
$\Rightarrow \sqrt{3} = \frac{x+4}{\sqrt{3}x}$
$\Rightarrow 3x = x + 4$
$\Rightarrow 2x = 4 \Rightarrow x = 2$
Thus, height of balloon $= x + 4 + 2 \Rightarrow 2 + 4 + 2 = 8m$

31. 

We know that tangent segments to a circle from the same external point are congruent
Now, we have
AE = AF,
BD = BE =6 cm and
CD = CF = 9 cm
Now,
$\text{Area}(\triangle ABC)$
$= \text{Area}(\triangle BOC) + \text{Area}(\triangle AOB) + \text{Area}(\triangle AOC)$
$\Rightarrow 54 = \frac{1}{2} \times BC \times OD + \frac{1}{2} \times AB \times OE + \frac{1}{2} \times AC \times OF$
$\Rightarrow 108 = 15 \times 3 + (6 + x) \times 3 + (9 + x) \times 3$
$\Rightarrow 36 = 15 + 6 + x + 9 + x$
$\Rightarrow 36 = 30 + 2x$
$\Rightarrow 2x = 6$
$\Rightarrow x = 3cm$
$\therefore AB = 6 + 3 = 9cm$ and
$AC = 9 + 3 = 12cm.$

OR

Given, AP and BP are tangents to a circle with centre O



$AP = BP$ [tangents from external point P]

$\therefore \quad \angle PAB = \angle PBA$ [Angles opposite to equal sides]

Now $\angle APB + \angle PAB + \angle PBA = 180^\circ$

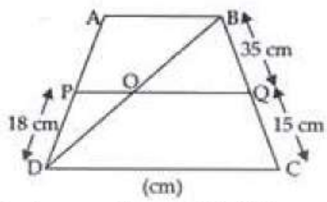
$\Rightarrow 60^\circ + 2\angle PAB = 180^\circ$

$\Rightarrow \quad \angle \mathrm{PAB} = 60^\circ$

$\therefore \triangle$APB is an equilateral triangle.

So, $AB = AP = 5cm$

Section D

32. 

In trapezium ABCD

AB || CD (Given)

PQ || DC (Given)

and PD = 18 cm, BQ = 35 cm and QC = 15 cm

To find: AD

∴ AB || CD || PQ ......(i)

In ΔBCD,

OQ || CD [From (i)]

$\therefore \frac{BO}{OD} = \frac{BQ}{QC}$ (ii) [By BPT]

Similarly, in ΔDAB,

PO || AB [From (i)]

$\therefore \frac{BO}{OD} = \frac{AP}{PD}$ (iii) [By BPT]

From (ii) and (iii)

$\frac{AP}{PD} = \frac{BQ}{QC}$

$\Rightarrow \frac{AP}{18} = \frac{35}{15}$

$\Rightarrow$ AP = $\frac{35}{15} \times 18 = 7 \times 6$

$\Rightarrow$ AP = 42 cm

∴ AD = AP + PD = 42 cm + 18 cm = 60 cm.

33. Let the original speed of the train be x km an hour

Then, the total speed taken by the train to travel a distance of 300 km at a uniform speed of x km an hour = $\frac{300}{x}$hours

Increased speed of the train = (x + 5) km an hour

Time taken by the train to travel a distance of 300 km at the increased speed = $\frac{300}{x+5}$ hours

According to the equation, $\frac{300}{x} - 2 = \frac{300}{x+5}$

$\Rightarrow \frac{300}{x} - \frac{300}{x+5} = 2 \Rightarrow 300\left(\frac{1}{x} - \frac{1}{x+5}\right) = 2$

$\Rightarrow \frac{1}{x} - \frac{1}{x+5} = \frac{2}{300} \Rightarrow \frac{1}{x} - \frac{1}{x+5} = \frac{1}{150}$

$\Rightarrow \frac{x+5-x}{x(x+5)} = \frac{1}{150} \Rightarrow \frac{5}{x^2+5x} = \frac{1}{150}$

$\Rightarrow x^2 + 5x = 750$

$\Rightarrow x^2 + 5x - 750 = 0$

Comparing with $ax^2 + bx + c = 0$

a = 1, b = 5, c = -750

Using the quadratic formula, $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

we get $= \frac{-5 \pm \sqrt{(5)^2-4(1)(-750)}}{2(1)}$

$= \frac{-5 \pm \sqrt{25+3000}}{2} = \frac{-5 \pm \sqrt{3025}}{2} = \frac{-5 \pm 55}{2}$

$= \frac{-5+55}{2}, \frac{-5-55}{2} = 25, -30$

x = - 30 is inadmissible as x is the speed of the train and speed cannot be negative.

$\therefore$ x = 25

Hence, the original speed of the train is 25 km/hr.

OR

Let the speed of the train be x km/hr for first 54 km and for next 63 km, speed is (x + 6) km/hr.

According to the question

$\frac{54}{x} + \frac{63}{x+6} = 3$

$\frac{54(x+6)+63x}{x(x+6)} = 3$

or, 54x + 324 + 63x = 3x(x + 6)

or, $117x + 324 = 3x^2 + 18x$

or, $3x^2 - 99x - 324 = 0$

or, $x^2 - 33x - 108 = 0$

or, $x^2 - 36x + 3x - 108 = 0$

or, x(x - 36) + 3(x - 36) = 0

(x - 36)(x + 3) = 0

x = 36

x = - 3 rejected.

(as speed is never negative)

Hence First speed of train = 36 km/h

34. The given data are:

Marks	Number of students
25 or more than 25	52
35 or more than 35	47
45 or more than 45	37
55 or more than 55	17
65 or more than 65	8
75 or more than 75	2
85 or more than 85	0

From above data we can calculate range data as following:

Marks	Number of students(f)
25 - 35	52 - 47 = 5
35 - 45	47 - 37 = 10

Marks	Number of students(f)
45 - 55	37 - 17 = 20
55 - 65	17 - 8 = 9
65 - 75	8 - 2 = 6
75 - 85	2 - 0 = 2
85 - 95	0

From table it is clear that maximum class frequency is 20 belonging to class interval 45 - 55

Modal class = 45 - 55

Lower limit (l) of modal class = 45

Class size (h) = 10

Frequency (f_1) of modal class = 20

Frequency (f_0) of class preceding modal class = 10

Frequency (f_2) of class succeeding the modal class = 9

Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 45 + \left(\frac{20-10}{2\times 20-10-9}\right) \times 10$

$= 45 + \frac{10}{21} \times 10$

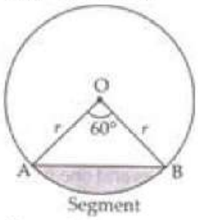
= 45 + 4.76

= 49.76

Therefore mode of data is 49.76

35. Area of minor segment = Area of sector – Area of $\triangle$OAB

In $\triangle$OAB,



$\theta = 60°$

OA = OB = r = 12 cm

$\angle B = \angle A = x$ [$\angle$s opp. to equal sides are equal]

$\Rightarrow \angle A + \angle B + \angle O = 180°$

$\Rightarrow x + x + 60° = 180°$

$\Rightarrow 2x = 180° - 60°$

$\Rightarrow x = \frac{120°}{2} = 60°$

$\therefore \triangle$OAB is equilateral $\triangle$ with each side (a) = 12 cm

Area of the equilateral $\triangle = \frac{\sqrt{3}}{4}a^2$

Area of minor segment = Area of the sector – Area of $\triangle$OAB

$= \frac{\pi r^2 \theta}{360°} - \frac{\sqrt{3}}{4}a^2$

$= \frac{3.14\times 12\times 12\times 60°}{360°} - \frac{\sqrt{3}}{4} \times 12 \times 12$

$= 6.28 \times 12 - 36\sqrt{3}$

$\therefore$ Area of minor segment = $(75.36 - 36\sqrt{3})$ cm^2.

OR

Let the radii of the two circular plots be r_1 and r_2, respectively.

Then, $r_1 + r_2 = 14$ [$\because$ Distance between the centres of two circular plots = 14 cm,given]....(i)

Also, Sum of Areas of the plots = 130π

$\therefore \pi r_1^2 + \pi r_2^2 = 130\pi \Rightarrow r_1^2 + r_2^2 = 130$ ...(ii)

Now, from equation (i) and equation (ii),

$\Rightarrow (14 - r_2)^2 + r_2^2 = 130$

$\Rightarrow 196 - 2r_2 + 2r_2^2 = 130$

$\Rightarrow 66 - 2r_2 + 2r_2^2 = 0$

Solving the quadratic equation we get,

$r_2 = 3$ or $r_2 = 11$,

but from figure it is clear that, $r_1 > r_2$

$\therefore r_1 = 11$ cm and $r_2 = 3$ cm

The value depicted by the farmers are of cooperative nature and mutual understanding.

Section E

36. **Read the text carefully and answer the questions:**

Kamla and her husband were working in a factory in Seelampur, New Delhi. During the pandemic, they were asked to leave the job. As they have very limited resources to survive in a metro city, they decided to go back to their hometown in Himachal Pradesh. After a few months of struggle, they thought to grow roses in their fields and sell them to local vendors as roses have been always in demand. Their business started growing up and they hired many workers to manage their garden and do packaging of the flowers.

In their garden bed, there are 23 rose plants in the first row, 21 are in the 2^{nd}, 19 in 3^{rd} row and so on. There are 5 plants in the last row.

(i) The number of rose plants in the 1^{st}, 2^{nd}, ... are 23, 21, 19, ... 5

$a = 23, d = 21 - 23 = -2, a_n = 5$

$\therefore a_n = a + (n - 1)d$

or, $5 = 23 + (n - 1)(-2)$

or, $5 = 23 - 2n + 2$

or, $5 = 25 - 2n$

or, $2n = 20$

or, $n = 10$

(ii) Total number of rose plants in the flower bed,

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2(23) + (10-1)(-2)]$

$S_{10} = 5[46 - 20 + 2]$

$S_{10} = 5(46 - 18)$

$S_{10} = 5(28)$

$S_{10} = 140$

OR

$S_n = 80$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 80 = \frac{n}{2}[2 \times 23 + (n-1) \times -2]$

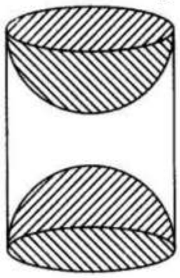
$\Rightarrow 80 = 23n - n^2 + n$

$\Rightarrow n^2 - 24n + 80 = 0$

$\Rightarrow (n - 4)(n - 20) = 0$
$\Rightarrow n = 4$ or $n = 20$
$n = 20$ not possible
$a_{20} = 23 + 19 \times (-2) = -15$
Number of plants cannot be negative.
$n = 4$

(iii) $a_n = a + (n - 1)d$
$\Rightarrow a_6 = 23 + 5 \times (-2)$
$\Rightarrow a_6 = 13$

37. **Read the text carefully and answer the questions:**
A carpenter used to make different kinds and different shapes of a toy of wooden material. One day a man came to his shop to purchase an article that has values as per his requirement. He instructed the carpenter to make the toy by taking a wooden block of rectangular shape with height 12 cm and width 9 cm, then shaping this block as a solid cylinder and then scooping out a hemisphere from each end, as shown in the given figure. The difference between the length of rectangle and height of the cylinder is 2 cm (Rectangle length > Cylinder height), and the difference between the breadth of rectangle and the base of cylinder is also 2 cm (Rectangle breadth > Cylinder base(diameter)).



(i) Given:
Length of rectangle = 12 cm
Width of rectangle = 9 cm
After scratching the rectangle into a cylinder,
Height of cylinder = 10 cm
Diameter of base = 7 cm
$\Rightarrow$ Radius of base = 3.5 cm
Volume of cylinder = $\pi r^2 h$
$= \frac{22}{7} \times 3.5^2 \times 10 = 385 \text{ cm}^3$

(ii) Given:
length of rectangle = 12 cm
width of rectangle = 9 cm
After scratching the rectangle into a cylinder,
height of cylinder = 10 cm
diameter of base = 7 cm
$\Rightarrow$ radius of base = 3.5 cm
Volume of wood scooped out = 2 $\times$ volume of hemisphere
$\Rightarrow$ Volume of wood scooped-out = $2 \times \frac{2}{3} \times \pi \times r^3$
$\Rightarrow$ Volume of wood scooped out = $\frac{4}{3} \times \frac{22}{7} \times (3.5)^3 = 179.66 \text{ cm}^3$

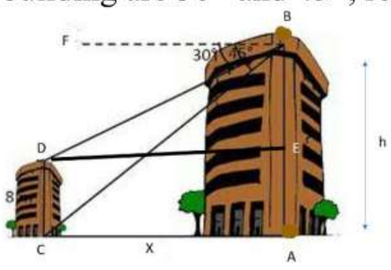
(iii) Given:
length of rectangle = 12 cm
width of rectangle = 9 cm
After scratching the rectangle into a cylinder,

height of cylinder = 10 cm
diameter of base = 7 cm
$\Rightarrow$ radius of base = 3.5 cm
Total surface area of the article
$= 2\pi(3.5)(10) + 2\left[2\pi(3.5)^2\right]$
$= 70\pi + 49\pi = 119\pi$
$= 119 \times \frac{22}{7} = 17 \times 22$
$= 374 \text{ cm}^2$

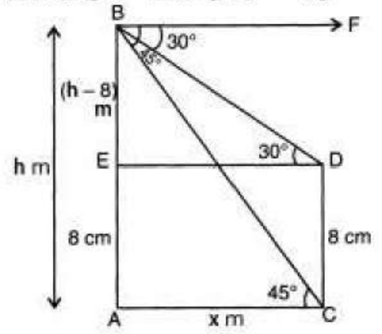
OR

Given:
length of rectangle = 12 cm
width of rectangle = 9 cm
After scratching the rectangle into a cylinder,
height of cylinder = 10 cm
diameter of base = 7 cm
$\Rightarrow$ radius of base = 3.5 cm
T.S.A of cylinder = $2\pi r(r + h)$
$\Rightarrow$ T.S.A of cylinder = $2 \times \frac{22}{7} \times 3.5\,(3.5 + 10) = 99 \text{ cm}^2$

38. **Read the text carefully and answer the questions:**
Basant and Vinod lives in a housing society in Dwarka, New Delhi. There are two building in their housing society. The first building is 8 meter tall. One day, both of them were just trying to guess the height of the other multi-storeyed building. Vinod said that it might be a 45 degree angle from the bottom of our building to the top of multi-storeyed building so the height of the building and distance from our building to this multi-storeyed building will be same. Then, both of them decided to estimate it using some trigonometric tools. Let's assume that the first angles of depression of the top and bottom of an 8 m tall building from top of a multi-storeyed building are 30° and 45°, respectively.



(i) Let h is height of big building, here as per the diagram.
AE = CD = 8 m (Given)
BE = AB - AE = (h - 8) m
Let AC = DE = x
Also, $\angle FBD = \angle BDE = 30^{\circ}$
$\angle FBC = \angle BCA = 45^{\circ}$



In $\triangle$ACB, $\angle A = 90^{\circ}$

$\tan 45^\circ = \frac{AB}{AC}$

$\Rightarrow$ x = h, ...(i)

In $\triangle$BDE, $\angle E = 90^\circ$

$\tan 30^\circ = \frac{BE}{ED}$

$\Rightarrow x = \sqrt{3}(h-8)$.(ii)

From (i) and (ii), we get

$h = \sqrt{3}h - 8\sqrt{3}$

h($\sqrt{3}$ - 1) = 8$\sqrt{3}$

h = $\frac{8\sqrt{3}}{\sqrt{3}-1} = \frac{8\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \frac{1}{2} \times (24 + 8\sqrt{3}) = \frac{1}{2} \times (24 + 13.84)$ = 18.92 m

(ii) Let h is height of big building, here as per the diagram.

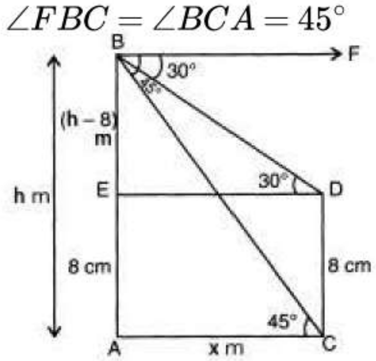
AE = CD = 8 m (Given)

BE = AB - AE = (h - 8) m

Let AC = DE = x

Also, $\angle FBD = \angle BDE = 30^\circ$

$\angle FBC = \angle BCA = 45^\circ$



In $\triangle$ACB, $\angle A = 90^\circ$

$\tan 45^\circ = \frac{AB}{AC}$

$\Rightarrow$ x = h, ...(i)

In $\triangle$BDE, $\angle E = 90^\circ$

$\tan 30^\circ = \frac{BE}{ED}$

$\Rightarrow x = \sqrt{3}(h-8)$.(ii)

From (i) and (ii), we get

$h = \sqrt{3}h - 8\sqrt{3}$

h($\sqrt{3}$ - 1) = 8$\sqrt{3}$

h = $\frac{8\sqrt{3}}{\sqrt{3}-1} = \frac{8\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \frac{1}{2} \times (24 + 8\sqrt{3}) = \frac{1}{2} \times (24 + 13.84)$ = 18.92 m

Hence height of the multistory building is 18.92 m and the distance between two buildings is 18.92 m.

OR

In $\triangle$BDE

cos $30^{o} = \frac{ED}{BD}$

$\Rightarrow BD = \frac{ED}{\cos 30^0}$

$\Rightarrow BD = \frac{\frac{8\sqrt{3}}{\sqrt{3}-1}}{\frac{\sqrt{3}}{2}} = \frac{16}{\sqrt{3}-1}$

$\Rightarrow BD = 8\left(\sqrt{3}+1\right)$ = 21.86 m

Hence, the distance between top of multistoried building and top of first building is 21.86 m.

(iii)In $\triangle$ABC

sin $45^\circ = \frac{AB}{BC}$

$\Rightarrow BC = \frac{AB}{\sin 45^0}$

$\Rightarrow BC = \frac{18.92}{\frac{1}{\sqrt{2}}}$

$\Rightarrow$ BC = 26.76 m

Hence the distance between top of multistoried building and bottom of first building is 26.76 m.